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THE EFFECT OF
QUEUE LENGTH AND
TIME OF DAY ON SERVICE TIME

A THESIS

Presented to

the Faculty of the Graduate Division

by

Robert Ernest Williams

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THE EFFECT OF

QUEUE LENGTH AND

TIME OF DAY ON SERVICE TIME.

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SUMMARY

The purpose of this study was to test the validity of the basic assumption in waiting line models that service time is independent of queue length and/or time of day. In addition, it was intended to develop mathematical expressions describing the correlations between these variables and to use these expressions where feasible in obtaining more reliable minimum cost solutions to an actual waiting line problem.

Sample data for the experiment was obtained by time study observations of service times for variable queue lengths and times of day at an aircraft plant blueprint crib. Data was also obtained on arrival rates to allow minimum cost solutions to the problem. The least squares statistical technique (utilizing the IBM-650 computer) was used on the experimental data to obtain mathematical descriptions of the correlations between the variables, and the "F" test was used for determining the best fitting mathematical relationships.

Finally, minimum cost solutions were calculated, first, considering and, then, not considering, the significant and usable relationships between these variables. A varying arrival rate was also considered in the latter solution. These minimum cost solutions were then compared to each other to see if more reliable minimum cost solutions had been obtained.

Within the limits imposed on this study it was concluded that:

(1) A strong quadratic correlation existed between service time and time of day and a weak linear correlation existed between service time and queue length.

(2) A cause and effect relationship was considered to exist between service time and time of day based on the large history of worker performance curves during a work shift and the results of this study.

(3) A cause and effect relationship was not considered to exist between service time and queue length but only an empirical relationship due principally to the simultaneous occurrence of slowest mean service time and highest arrival rate in the early hours of the shift.

(4) The minimum cost solutions were improved significantly by a consideration of the effects of time of day on the service time and a consideration of the varying arrival rates.

(5) An incorrect minimum cost solution would have resulted if the varying service and arrival rates for the different times of day had not been considered.

(6) The arrival time distributions were found to be hyper-exponential and the service time distributions were found to be Erlang.

It was recommended, considering the results as well as the limitations of this study, that investigators of waiting line problems, utilizing human-controlled service mechanisms, consider carefully the effects of time of day on service time and also varying arrival rates to avoid possible incorrect minimum cost solutions.

CHAPTER I

INTRODUCTION

Is the assumption valid that service time is independent of queue length and/or time of day in waiting line models? This assumption is basic in the present mathematical approach to the solution of waiting line problems; however, an investigation of the possible dependence of service time on these factors seems in order. The purpose of this investigation, therefore, is to develop mathematical expressions for the relationship between service time and queue length and/or time of day, and to use these expressions where feasible in obtaining more reliable minimum cost solutions to an actual waiting line problem, using the existing theory of queueing models.

A brief discussion of queueing theory (including its definition, application, and history) is given at this point to fully describe the problem.

The study of waiting lines and their behavior is descriptively called waiting line theory or queueing theory. Queueing occurs when the demands for service are greater than the availability of such service, and it becomes necessary to postpone these demands by a system of queueing or marshalling.

Queueing theory is primarily used to minimize the total cost of "customer" waiting and service facility idleness by choosing the optimum combination of controllable factors that results in a minimum cost operation.

The development and extension of waiting line theory to the solution of industrial problems offer to management an important aid for making rational and economic decisions. When a waiting-line process has been correctly described and costs have been properly specified, waiting-line analysis provides a powerful tool for management. In a large number of problems the same mathematical model and formulas apply if there are random arrival and service times, and in each case these formulas can be used to predict the operational situation and to modify it for best results.

The essential components of a queueing situation are described as follows:

(1) Input-the distribution of time intervals between the arrival of customers at the point of service.

(2) Queue-discipline-the manner in which customers are selected for service.

(3) Service-mechanism-the distribution of individual customer service times.

Although the term "queueing theory" is relatively new, actual work in the field began some fifty years ago. The pioneer investigator was Erlang, who in 1908 did fundamental work on telephone switching for the Copenhagen Telephone Company. Since that time, most of the work in queueing theory has been conducted in Great Britain, and more recently in America.

Waiting line theory can be applied to most situations involving random elements of input and output at one or more points of service. Customer waiting lines form in restaurants and stores, ships wait in harbors for dock space, airplanes stack up over airports waiting for an opportunity to land, manufactured parts pile up in production lines waiting for the next step in assembly, long distance calls wait for clear trunk lines, machinery breaks down and waits for maintenance crews, and sales slips wait to be posted to customers' accounts by a bookkeeper. Valid predictions, however, can only be made from an accurate description of the components of the queueing situation and a correct appraisal of the model parameters.

One basic assumption in queueing theory models is that individual service times are statistically independent and that the mean service rate is constant. This investigation analyzes a typical queueing situation at a blueprint crib in an aircraft manufacturing plant so that the "service times" can be compared with other factors (specifically queue length and/or time of day) to test the assumption that service time is independent of these factors. The ultimate goal will be to derive mathematical expressions for the relationship between service time and queue length and/or time of day and to use these expressions to quantitatively define the appropriate model necessary to obtain improved minimum cost solutions to a waiting line problem.

The present queueing theory approach to the problem would minimize the cost of this operation by establishing mathematical formulas to describe the service and arrival time distributions. These formulas would in turn be used to establish the optimum number of crib attendants to minimize the total cost of the blueprint crib operation.

If it is determined in this study, however, that service times are not independent of queue length and/or time of day, this effect on the "minimum cost" solution will be investigated. Modifications to the conventional queueing theory approach will be suggested if improved minimum cost solutions seem possible. The least squares statistical technique will be used on the experimental data to derive these mathematical relationships and the "F" test will be used to determine the significance of the relationships.

A survey of queueing theory literature revealed that no specific description of the relationship between service time and queue length and/or time of day is readily available. Several writers have mentioned the possible effect of queue length on the service mechanism and the effects of time of day on worker performance during a work shift, but none of them have described these effects quantitatively. Bailey (1) makes the statement:

> We shall further assume that the consultation times of successive patients are distributed independently of one another, although in practice this may not be entirely true. There may easily be a tendency of self regulation, so that consultation times are liable to increase if there are few patients waiting, but decrease if there is a long queue.

Marshall (2) stated, "There is a natural tendency to serve more rapidly if there is a long queue." Lindley (3) said, "Sometimes input and service times are not independent of each other. Both might be influenced by the size of the queue." Welch (4) noted, "The length of the queue itself had an appreciable effect on the speed at which a doctor worked; a long queue might make a doctor hurry, or beyond a certain point might cause him to slow down in despair." Smith (5) stated:

> If the amount of work done is estimated for each hour of the working day the results can be expressed graphically, and for a number of processes there is the same general trend, vis. a gradual rise in output till a period of stability is reached, which lasts for a varying period, and then a decrease towards the end of the period of work.
>
> Owing to the shape of this curve when graphed, the curve is known as the saddle-back curve. It is characteristic of both manual and mental work, if there is no change except the passing of hours. It has been described as showing "a sluggish start before the worker is warmed up, a rise as he gets in stride, a flagging, and a final falling off in the last hour."

At present then, mathematical expressions of the relationship between service times and queue length and/or time of day do not seem to exist.

CHAPTER II

EXPERIMENTAL ENVIRONMENT AND DATA COLLECTION

This waiting line study was made at Blueprint Substation Number One, Lockheed Aircraft Corporation, Marietta, Georgia. The line was composed of aircraft production workers who obtained blueprints from this crib frequently as a part of their routine work. A brief description of this operation follows:

A supervisor gives an assignment to a worker and presents him with a planning work sheet that shows the engineering blueprints required to do the assignment. The worker, after determining which prints are required, goes to the blueprint crib, fills out a Print Assignment Slip, and hands it to the crib attendant. The attendant pulls the blueprints from the files and returns them to the worker. The worker then proceeds to his work area and uses the blueprints until completion of the assignment or until the end of the shift, at which time he returns the blueprints to a "drop box" outside the crib. This is done to assure that up-to-date blueprints will be available for the next shift since engineering changes are constantly being released to supplement the original blueprint. These supplements are stapled to the original blueprints by the crib attendants during the periods when there are no customers to be served. .

The complexity of the blueprint service operation was noted to be variable because of the varying number of blueprints required for different jobs. No important variation in this complexity was observed, however, with respect to the time of day.

The longest waiting lines and an above-average arrival rate have also been observed at this crib during the first hour of each shift. The main reason for this is that all prints have been turned in by the previous shift and because production work cannot proceed on the new shift (on blueprint controlled jobs) without these blueprints.

The crib is operated two shifts per day - by two clerks on the day shift (7:00 A. M. to 3:45 P. M.) and by two clerks on the swing shift (3:45 P. M. to 12:30 A. M.). All production workers are served at a single window and queue up in a single line. For the purpose of this study no changes were made to the blueprint crib layout, the method of service, the number of clerks, or the arrival rate.

Observations were made at the blueprint crib with a stopwatch for an eight hour period on the day shift to obtain the following data: (1) the lengths of time required to serve individual workers, (2) the queue lengths relative to each service time, (3) the time of day relative to each service time and (4) the lengths of time between arrivals. These observations were made by utilizing a continuous time study technique that showed for each worker his arrival time at the end of the queue, his arrival time at the service window, and his exit time leaving the service window. By making the appropriate subtractions of these timed observations, data showing the arrival rate, the waiting times, the service times, the queue lengths for corresponding service times, and the times of day for corresponding service times can be obtained. Appendix II shows the basic data for an eight hour period which was used for this study. The study period was considered typical of other days. Appendix V shows a sample form used in the collection of this data.

CHAPTER III

PROCEDURE

The basic data shown in Appendix II was first used to construct two dimensional scatter diagrams showing the following general relationships between the variables.

(1) Mean service time versus queue length

(2) Mean service time versus time of day

These scatter diagrams are shown in Figs. 1 and 2 in Chapter IV. The relationship between arrival rates and time of day was calculated and is shown in histogram form in Fig. 4.

The least squares statistical technique was next applied to the basic data to develop mathematical expressions that would best describe the correlations indicated by the scatter diagrams. Of necessity, the IBM-650 computer was used to make the least squares calculations from the input data shown in Appendix II. In addition to giving estimates of the regression coefficients, the standard computer routine yielded "F" test results on the significance level of each of the regression coefficients developed.

Finally, minimum cost solutions to the waiting line problem were calculated. The first solution was calculated without, and the second solution with, consideration of the significant and usable relationships between the variables. The second solution also considered the varying arrival rates during the day. These two minimum cost solutions were then compared to each other and to the actual cost of this crib's present operation. This comparison is presented in Chapter IV, and the detailed calculation of the solutions is shown in Appendix IV.

CHAPTER IV

DISCUSSION OF RESULTS

The results of the investigation show a strong quadratic correlation between service time and the time of day at the 0.01 level of significance for a full day. Also revealed was a weak linear correlation between service time and queue length at the 0.05 level of significance for a full day. These correlations are depicted below in Figs. 1 and 2 in the form of scatter diagrams and the plotted least squares regression equations. The significance level of these equations for the full day and of other equations developed for smaller time periods during the day are shown in Appendix III.

The curve depicted in Fig. 2 showing the strong relationship between service time and time of day bears a significant resemblance to the performance curve during the day for workers engaged in repititious operations. A description by Smith (5) of this performance curve has been presented in Chapter I of this paper. The curve indicates a slow start in the early morning hours and a gradual improvement in performance until mid-day, when performance reaches its peak. Performance then gradually decreases to another low period in late afternoon nearly equal to the early morning low period. This relationship supports the almost universally accepted concepts of worker performance during a work shift. Therefore, a cause and effect relationship was considered to exist between service time and time of

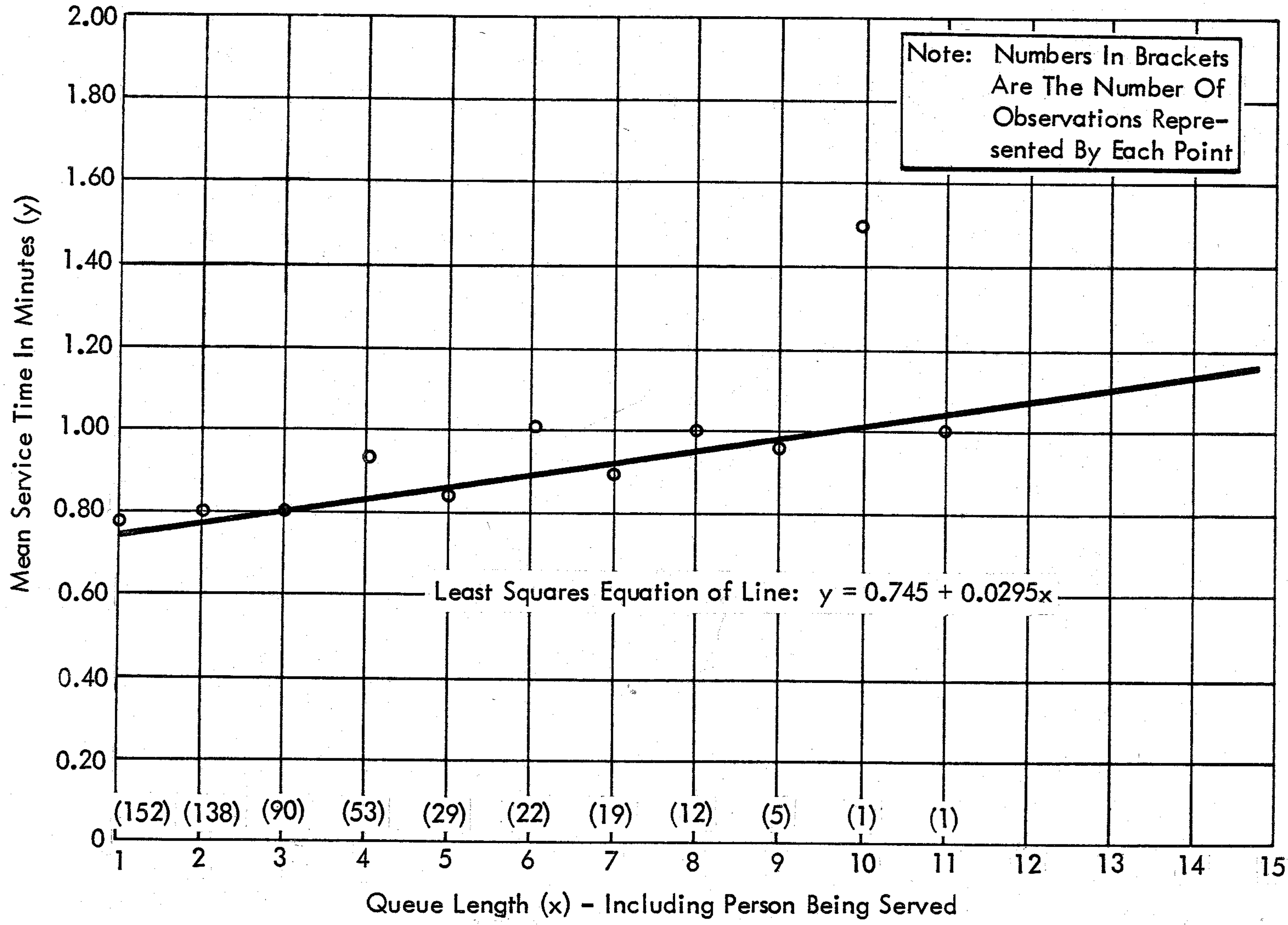

Figure 1 Correlation Of Service Time And Queue Length

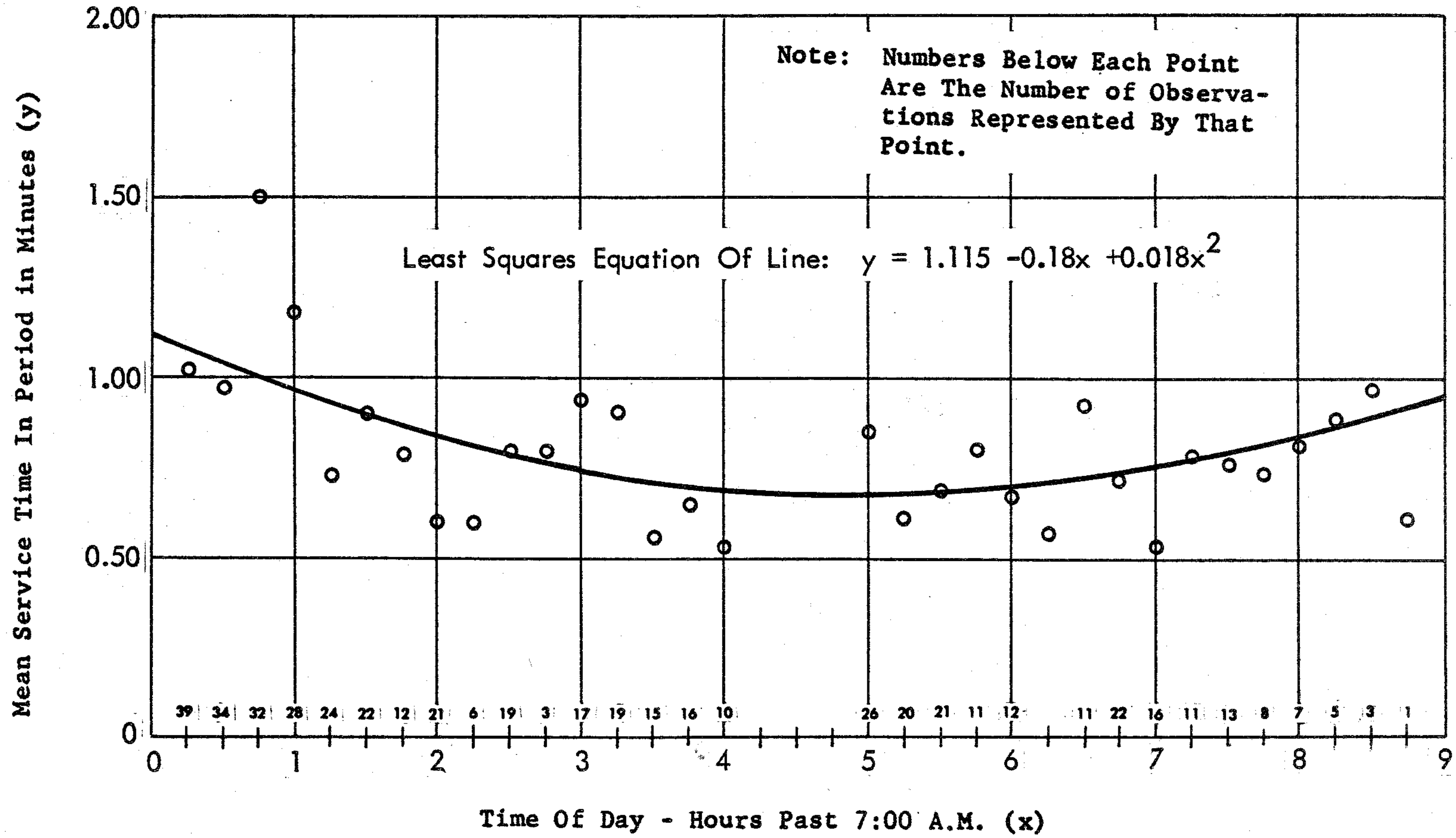

Figure 2 Correlation of Service Time And Time Of Day

day in this waiting line problem.

The curve shown in Fig. 1 depicting the weak linear correlation between service time and queue length is considered to be only an empirical relationship (not cause and effect) due principally to the simultaneous occurrence of slowest mean service time and highest arrival rate at the beginning of the shift.

Therefore, for the purpose of calculating minimum cost solutions, the effect of queue length on service time was not considered. The day was divided into the two periods indicated by Figs. 3 and 4 in histogram form (7:00 A. M. to 8:00 A. M. and 8:00 A. M. to 3:45 P. M.) to give adequate consideration for the slow service and high arrival rates during the first hour of the shift. The effect of these relationships on the minimum cost solutions, as shown in Appendix IV is summarized by noting that two attendants for a complete day yielded the minimum cost solution when service time dependency and varying arrival rates were not considered. Whereas, the alternate solution to this problem, considering the varying arrival rate and the effect of time of day on service time, yielded a minimum cost solution of four attendants from 7:00 A. M. to 8:00 A. M. and of two attendants from 8:00 A. M. to 3:45 P. M.

A fallacy exists, however, in the first minimum cost solution which disregards the effect of time of day on service time and the varying arrival rate. This fallacy is apparent in that composite arrival and service rates (for the complete day) were used rather than true arrival and service rates (for the sub-periods). When the true arrival and service rates are applied to the period from 7:00 A. M. to

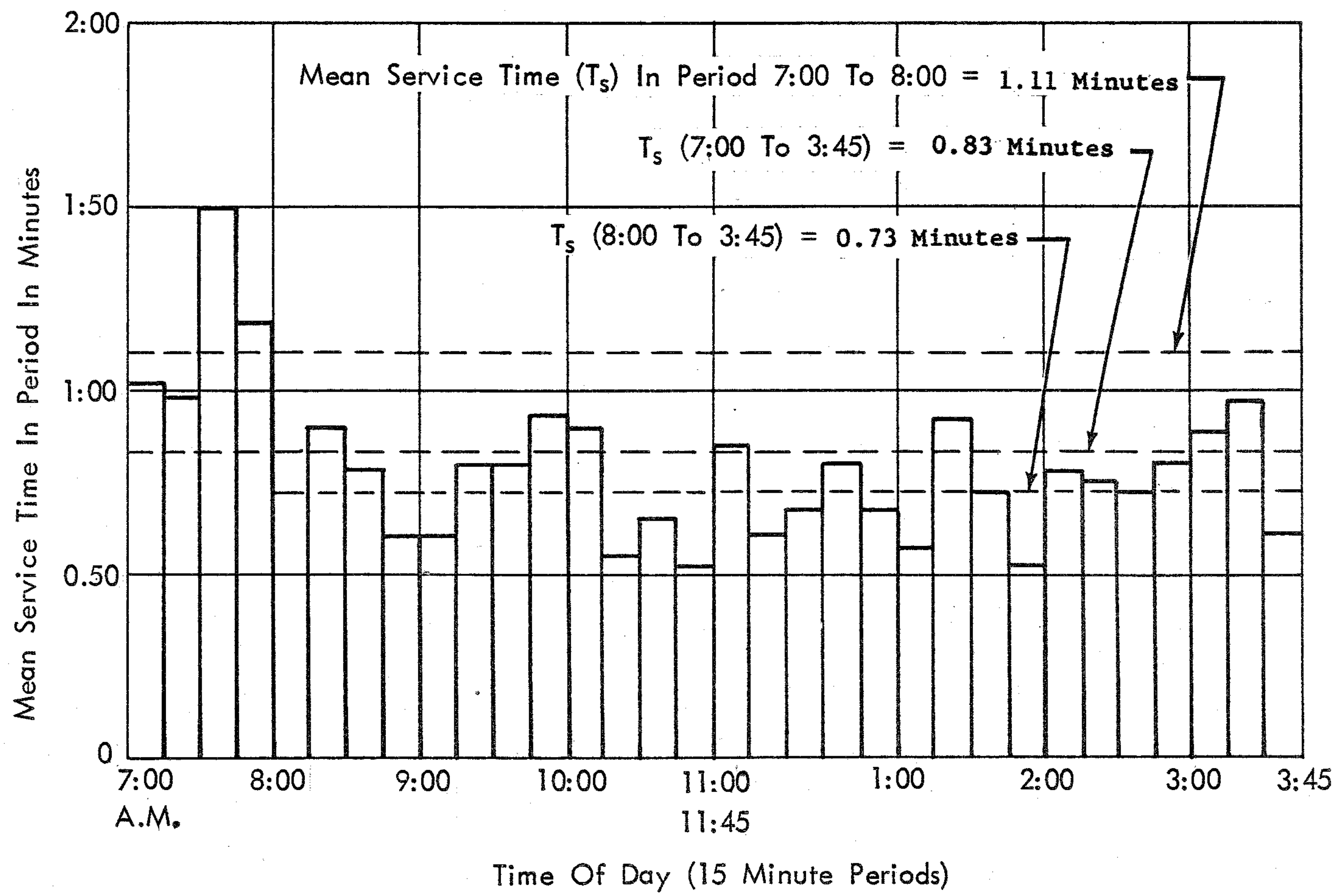

Figure 3 Histogram - Mean Service Times During Day

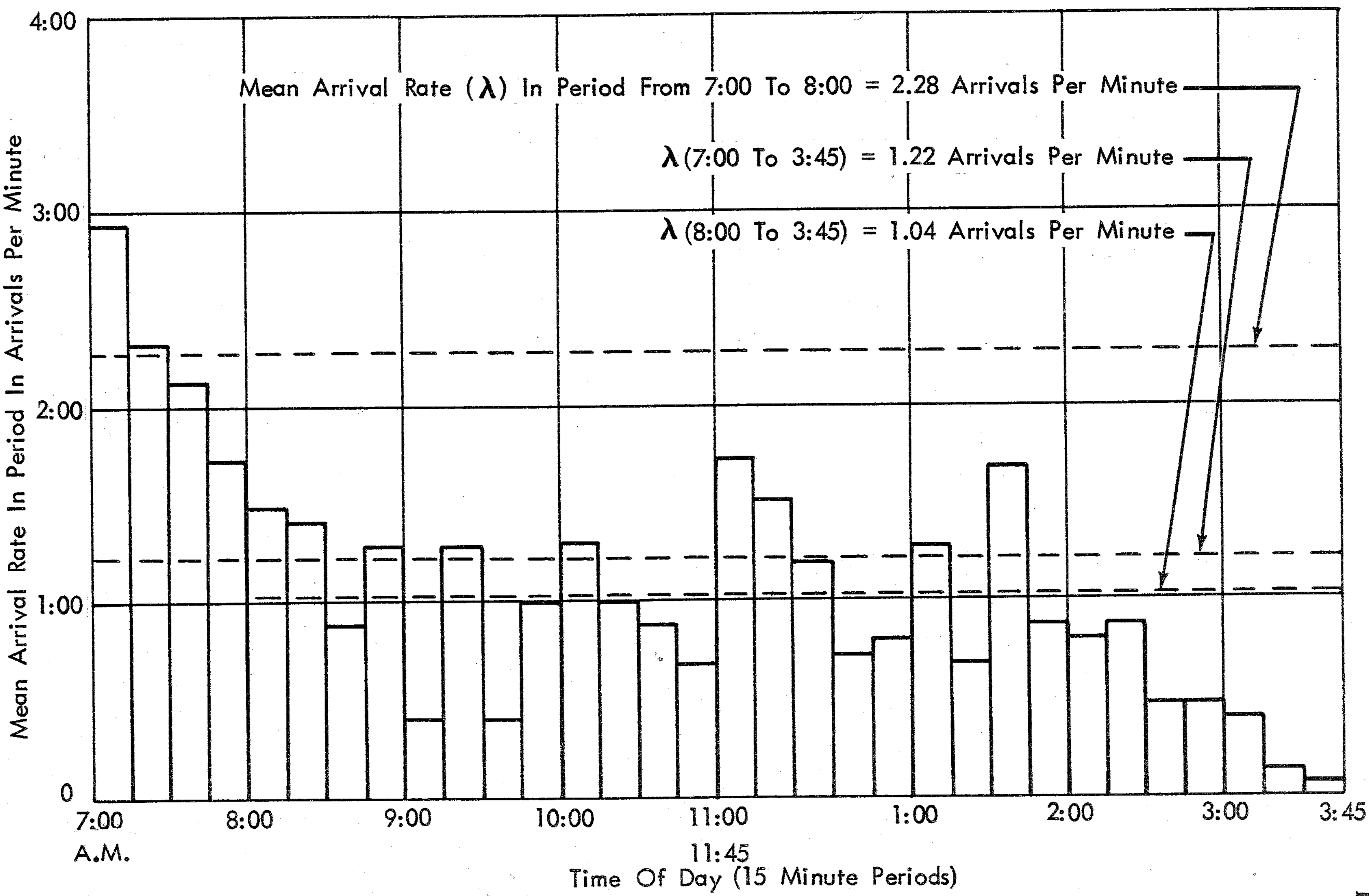

Figure 4 Histogram - Mean Arrival Rates During Day

8:00 A. M. with the recommended two attendants in service, an infinite queue and consequently an infinite cost results from waiting. Actually the length of this line would not become infinite because only 522 arrivals occur during the day, but its length would become exceedingly long and its waiting cost excessive. The calculation of the actual cost of the two-attendant operation for the full day has not been performed, since it is beyond the scope of this paper. Let it suffice to say that the cost of such operation would be considerably higher than the alternate solution of utilizing four attendants from 7:00 A. M. to 8:00 A. M. and two attendants from 8:00 A. M. to 3:45 P. M., and also that the engineer who recommended and initiated a two-attendant plan for the full day would be extremely uncomfortable in observing this waiting line in the early hours of the first day.

However, a comparison of costs is shown in Appendix IV between the alternate minimum cost solution (utilizing four clerks from 7:00 A. M. to 8:00 A. M. and two clerks for the balance of the day) and the actual cost of the present crib operation (utilizing three clerks all day). A cost reduction of $3630 per year would be realized in the crib operation on the day shift using the principles set forth in this paper. Similar savings could be expected for other cribs and other shifts.

The best available queueing model which assumed Poisson arrivals, exponential service, an infinite queue, and multiple channels was used in obtaining the minimum cost solutions summarized above and calculated in Appendix IV. These minimum cost solutions were also calculated during this investigation using another model as follows:

1. Erlang service time distributions
2. Poisson or hyper-exponential arrival distributions
3. Infinite queue
4. Single channel with varying service rates to simulate varying number of attendants

The same solutions were obtained, however, as those from the Appendix IV model, with the same number of attendants being required for minimum cost. The latter solutions are not shown in this paper because the model described in Appendix IV is more appropriate - and both models provide the same solutions.

Even more appropriate models for the minimum cost solutions to these problems would have been varying combinations of the following factors shown in the Figures in Appendix IV.

1. Erlang "k" service time distributions
2. Poisson or hyper-exponential arrival distributions
3. Infinite queue
4. Multiple channels

These models, however, were not available and the development of these models was beyond the scope of this paper. It is not believed that the use of these more appropriate models, if they had been available, would have significantly improved the solutions to these problems. This reasoning is based on the fact that all of the experimental situations were combinations of Erlang "k" and hyper-exponential "j" distributions. The effects of the combination of these distributions normally cancel each other and, therefore, make appropriate the use of Poisson and exponential distributions.

CHAPTER V

CONCLUSIONS

For the eight hour (day-shift) period sampled, for this blueprint crib operation, and for the various other limitations stated elsewhere in this paper it is concluded that:

(1) A strong quadratic correlation exists between service time and time of day and a weak linear correlation exists between service time and queue length.

(2) A cause and effect relationship is considered to exist between service time and time of day based on the large history of worker performance curves during a work shift and the results of this study.

(3) A cause and effect relationship is not considered to exist between service time and queue length but only an empirical relationship due principally to the simultaneous occurrence of slowest mean service time and highest arrival rate.

(4) The minimum cost solutions were improved significantly by a consideration of the effects of time of day on the service time and a consideration of the varying arrival rates.

(5) An incorrect minimum cost solution would have resulted if the varying service and arrival rates for the different times of day had not been considered.

(6) The arrival time distributions were found to be hyper-exponential and the service time distributions were found to be Erlang.

CHAPTER VI

RECOMMENDATIONS

It is recommended, considering the results as well as the limitations of this study, that investigators of waiting line problems, utilizing human-controlled service mechanisms, consider carefully the effects of time of day on service time and also varying arrival rates to avoid possible incorrect minimum cost solutions.

Areas for future investigation on problems related to this waiting line study are noted in concluding this paper.

(1) Mathematical models which can be used jointly for varying combinations of Erlang service time distributions and hyper-exponential arrival time distributions for multiple channel operations are needed to provide better analytical models of actual queueing situations.

(2) An investigation of other service operations would be useful to determine whether similar service time to time of day relationships exist in these operations. A general relationship between these variables might be developed that would apply to most human-controlled service operations.

(3) Random observations of this blueprint crib service operation at varying times of day over a period of several months would be useful in validating a conclusion that this service time to time of day relationship exists on a long term basis.

APPENDIX I

LIST OF SYMBOLS USED

$A_o(t)$ -- Arrival time distribution function

C_1 -- Cost of waiting line operation, total dollars, utilizing one clerk

C_2 -- Cost of waiting line operation, total dollars, utilizing two clerks, etc.

$D_m(x)$ -- Poisson integral function

$e_m(x)$ -- Poisson density function

j -- Parameter in hyper-exponential distribution

k -- Number of stages in an Erlang operation

L -- Mean total number in system

L_q -- Mean number in queue

M -- Number of service channels

$S_o(t)$ -- Service-time distribution function

S_t -- Cost of service, total dollars

T_a -- Mean time between arrivals

T_s -- Mean service time

t -- Time

W -- Mean waiting time, total

W_t -- Cost of waiting, total dollars

λ -- Mean arrival rate

μ -- Mean service rate

ρ -- Utilization parameter

APPENDIX II

Table 1. Data for Least Squares and Optimum Cost Calculations

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
1	1.2	6	0.0	2.4
2	1.1	5	0.0	0.0
3	0.4	4	0.0	0.0
4	2.3	3	0.0	0.0
5	1.2	2	0.0	0.0
6	1.7	1	0.0	0.0
7	1.5	1	0.0	1.8
8	1.1	5	0.0	0.8
9	1.2	4	0.0	0.0
10	1.3	4	0.1	0.0
11	1.1	5	0.1	0.0
12	1.7	4	0.1	0.0
13	1.5	4	0.1	1.0
14	1.4	4	0.1	0.3
15	1.6	4	0.1	0.0
16	1.0	8	0.1	0.4
17	1.0	7	0.1	0.5
18	1.4	6	0.1	0.2
19	1.0	8	0.1	0.4
20	0.7	7	0.1	0.0
21	0.9	7	0.1	0.0
22	0.6	7	0.1	0.2
23	0.9	7	0.1	0.0
24	0.4	7	0.1	0.7
25	1.0	6	0.1	0.3
26	0.3	8	0.1	0.0
27	0.3	7	0.1	0.6
28	0.2	6	0.1	0.2
29	0.5	5	0.1	0.3
30	0.7	5	0.1	0.5
31	0.7	5	0.1	0.4
32	1.0	4	0.1	0.0
33	0.9	6	0.2	0.0
34	0.7	5	0.2	0.5
35	0.8	4	0.2	0.1
36	0.6	3	0.2	0.4
37	1.1	3	0.2	0.0

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
38	1.2	4	0.2	0.0
39	1.3	3	0.2	1.2
40	1.1	2	0.2	0.2
41	0.6	4	0.2	0.1
42	0.8	4	0.2	1.0
43	1.7	3	0.2	0.1
44	1.7	3	0.2	0.1
45	1.7	2	0.2	0.3
46	0.7	6	0.2	0.7
47	0.3	5	0.2	0.4
48	0.8	4	0.2	0.3
49	0.8	3	0.2	0.1
50	1.0	2	0.2	0.2
51	0.9	2	0.3	0.1
52	1.5	3	0.3	1.3
53	1.7	2	0.3	0.3
54	1.6	2	0.3	0.1
55	0.8	2	0.3	0.6
56	1.0	2	0.3	0.9
57	1.0	2	0.3	1.1
58	1.5	1	0.3	0.3
59	0.7	3	0.3	0.9
60	0.8	2	0.3	0.2
61	1.0	5	0.4	0.0
62	1.3	4	0.4	1.1
63	0.9	5	0.4	0.0
64	0.7	8	0.4	0.4
65	0.7	7	0.4	0.0
66	0.7	6	0.4	1.3
67	0.7	5	0.4	0.0
68	0.8	5	0.4	0.4
69	0.8	6	0.4	0.3
70	0.6	5	0.4	0.0
71	0.4	4	0.4	0.0
72	0.8	5	0.4	0.9
73	1.0	4	0.4	0.1
74	4.6	3	0.4	0.0
75	4.4	2	0.4	0.8
76	1.0	11	0.5	0.0
77	1.5	10	0.5	1.2
78	1.8	9	0.5	0.5
79	1.9	8	0.5	0.3

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
80	1.7	8	0.5	0.2
81	1.6	8	0.5	0.0
82	1.5	7	0.5	0.1
83	1.1	9	0.5	0.0
84	1.1	8	0.5	0.7
85	0.6	7	0.5	0.2
86	0.9	6	0.5	0.5
87	0.5	6	0.6	1.7
88	0.7	5	0.6	0.8
89	0.5	6	0.6	0.3
90	0.7	7	0.6	0.2
91	1.1	6	0.6	0.3
92	1.3	5	0.6	1.2
93	1.8	4	0.6	0.1
94	1.9	6	0.6	0.1
95	2.0	6	0.6	0.1
96	0.7	8	0.6	0.1
97	0.9	7	0.6	1.1
98	1.3	6	0.6	0.1
99	1.4	5	0.6	0.2
100	1.2	7	0.6	0.2
101	1.1	6	0.6	0.3
102	1.2	6	0.6	0.7
103	2.5	6	0.6	0.5
104	2.0	7	0.7	1.1
105	1.6	6	0.7	0.0
106	0.8	5	0.7	0.0
107	1.2	4	0.7	0.9
108	1.3	4	0.7	0.3
109	1.3	4	0.7	0.2
110	1.0	4	0.7	0.4
111	1.1	3	0.7	2.1
112	0.6	3	0.7	0.6
113	0.8	2	0.7	0.5
114	0.8	3	0.7	0.4
115	1.2	2	0.7	0.8
116	0.6	3	0.7	0.0
117	0.5	2	0.8	0.8
118	0.5	2	0.8	0.0
119	0.8	1	0.8	0.9
120	1.5	5	0.8	2.3
121	1.2	4	0.9	2.2

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
122	1.0	3	0.9	0.0
123	1.5	2	0.9	0.0
124	1.4	1	0.9	0.0
125	1.6	4	0.9	2.6
126	1.3	3	0.9	0.4
127	0.9	2	0.9	0.0
128	0.4	2	0.9	0.0
129	0.6	5	0.9	1.4
130	1.0	4	0.9	0.4
131	0.5	3	0.9	0.2
132	0.9	2	0.9	0.0
133	0.9	4	0.9	0.0
134	0.9	3	1.0	1.0
135	0.9	2	1.0	0.0
136	0.7	2	1.0	0.0
137	1.6	2	1.0	0.8
138	1.2	4	1.0	0.2
139	1.0	3	1.0	0.8
140	1.0	3	1.0	0.2
141	0.5	3	1.0	0.1
142	0.4	3	1.0	0.7
143	0.4	3	1.0	0.7
144	0.5	2	1.0	0.3
145	0.5	3	1.0	0.7
146	0.3	2	1.0	1.2
147	0.6	1	1.0	0.5
148	0.6	2	1.1	0.9
149	0.6	2	1.1	0.3
150	0.3	1	1.1	0.5
151	1.1	1	1.1	0.9
152	0.6	2	1.1	1.4
153	1.0	1	1.1	0.4
154	0.5	1	1.1	1.9
155	1.0	2	1.2	0.5
156	0.7	4	1.2	1.0
157	0.3	3	1.2	1.2
158	1.1	2	1.2	0.1
159	0.7	1	1.2	0.1
160	2.3	5	1.2	3.2
161	0.7	9	1.3	0.5
162	0.5	9	1.3	0.3
163	0.7	9	1.3	0.0
164	0.7	8	1.3	0.1
165	0.8	7	1.3	0.9

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
166	0.9	7	1.3	0.0
167	0.9	6	1.3	0.2
168	1.0	5	1.3	0.0
169	0.4	5	1.3	0.4
170	0.6	4	1.3	0.9
171	0.4	3	1.3	0.3
172	2.0	2	1.3	0.6
173	0.6	4	1.4	0.5
174	1.6	3	1.4	2.0
175	0.8	3	1.4	0.0
176	1.8	2	1.4	0.6
177	0.2	3	1.4	1.2
178	0.5	2	1.4	1.3
179	0.7	1	1.4	0.4
180	1.4	1	1.4	0.8
181	1.3	2	1.5	2.5
182	0.6	1	1.5	1.0
183	0.5	2	1.5	1.6
184	0.7	1	1.5	0.4
185	0.7	3	1.6	2.8
186	0.5	2	1.6	0.2
187	0.5	2	1.6	0.2
188	0.8	1	1.6	2.6
189	0.4	2	1.6	0.7
190	1.3	1	1.6	0.8
191	0.8	1	1.7	2.4
192	1.0	2	1.7	1.1
193	0.4	3	1.7	0.2
194	0.4	2	1.7	1.0
195	0.9	2	1.7	0.0
196	0.4	2	1.7	1.0
197	0.6	1	1.7	0.5
198	0.7	1	1.8	1.0
199	0.4	3	1.8	1.4
200	0.4	2	1.8	0.0
201	1.1	1	1.8	0.4
202	0.7	3	1.8	2.1
203	0.2	3	1.8	0.0
204	0.3	2	1.8	0.0
205	0.4	3	1.8	0.4
206	0.5	4	1.8	0.7
207	1.0	3	1.8	0.0
208	0.7	4	1.9	0.5

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
209	0.7	3	1.9	0.0
210	0.6	2	1.9	0.9
211	0.8	1	1.9	0.0
212	0.4	1	1.9	2.3
213	0.3	1	2.0	4.2
214	0.4	1	2.0	0.8
215	0.8	1	2.0	2.8
216	0.3	1	2.0	1.2
217	0.5	1	2.1	2.0
218	1.3	1	2.2	4.1
219	0.2	3	2.2	4.2
220	0.4	2	2.2	0.5
221	0.9	2	2.2	0.0
222	1.7	3	2.2	0.6
223	0.9	4	2.3	0.4
224	0.8	3	2.3	0.0
225	0.8	4	2.3	1.2
226	1.0	3	2.3	0.2
227	1.4	2	2.3	0.8
228	1.3	4	2.3	0.1
229	1.0	3	2.3	1.1
230	0.9	2	2.3	0.0
231	0.7	2	2.3	0.0
232	0.5	1	2.3	1.2
233	0.5	1	2.3	1.8
234	0.8	1	2.4	1.1
235	0.6	1	2.4	0.6
236	0.5	2	2.4	0.7
237	0.4	1	2.4	0.2
238	0.7	1	2.6	0.0
239	0.4	1	2.6	0.6
240	0.5	1	2.7	1.5
241	0.9	2	2.7	1.2
242	0.8	3	2.7	0.3
243	0.5	2	2.7	1.1
244	0.3	1	2.7	0.3
245	1.4	1	2.8	3.0
246	0.7	2	2.8	1.4
247	1.0	1	2.8	0.0
248	0.7	2	2.8	2.3
249	0.7	1	2.8	0.3
250	1.0	2	2.9	1.4
251	0.8	1	2.9	1.6

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
252	0.8	5	2.9	0.5
253	1.2	4	2.9	0.7
254	1.5	3	2.9	0.0
255	1.3	4	2.9	0.3
256	1.3	3	2.9	0.0
257	1.3	2	2.9	1.5
258	0.5	2	2.9	0.0
259	0.5	5	3.0	1.8
260	0.5	4	3.0	0.2
261	0.5	3	3.0	0.0
262	0.7	2	3.0	0.0
263	1.8	1	3.0	1.0
264	1.1	2	3.0	1.0
265	0.8	2	3.0	0.2
266	0.8	1	3.0	1.4
267	0.2	2	3.0	0.7
268	0.7	1	3.0	0.5
269	3.3	1	3.1	2.2
270	0.9	4	3.1	1.1
271	1.4	3	3.1	0.5
272	1.4	2	3.1	1.0
273	1.4	1	3.1	0.8
274	0.2	2	3.2	1.6
275	0.4	1	3.2	0.4
276	0.3	1	3.2	0.9
277	0.5	3	3.2	1.1
278	0.2	3	3.2	0.1
279	0.6	2	3.2	0.3
280	0.2	2	3.2	0.5
281	0.2	2	3.2	1.0
282	0.2	1	3.2	0.3
283	0.3	1	3.3	0.6
284	0.6	3	3.3	1.4
285	0.7	2	3.3	0.4
286	2.6	1	3.3	0.0
287	0.5	2	3.4	4.2
288	0.4	2	3.4	0.0
289	0.4	1	3.4	1.1
290	0.6	2	3.4	1.3
291	0.2	2	3.4	0.3
292	0.4	3	3.4	0.6
293	0.7	2	3.4	2.0
294	0.2	1	3.5	1.1

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
295	1.0	1	3.5	1.3
296	0.8	2	3.5	2.0
297	1.2	1	3.5	0.1
298	0.7	2	3.6	3.2
299	0.7	1	3.6	0.6
300	0.2	4	3.6	1.0
301	0.5	3	3.6	0.2
302	0.4	3	3.6	0.2
303	0.9	2	3.6	0.3
304	0.9	1	3.6	1.9
305	0.6	3	3.7	1.6
306	0.6	2	3.7	0.3
307	0.6	1	3.7	0.1
308	0.4	1	3.7	2.7
309	0.6	2	3.8	2.7
310	0.6	1	3.8	0.0
311	0.7	2	3.8	1.0
312	0.7	2	3.8	0.3
313	0.5	2	3.8	0.3
314	0.5	1	3.8	0.7
315	0.6	1	3.9	3.4
316	0.4	1	3.9	1.8
317	0.3	1	3.9	1.7
318	1.7	3	4.7	0.0
319	1.2	2	4.7	0.0
320	2.6	1	4.7	2.0
321	1.5	1	4.7	1.7
322	0.6	2	4.8	0.1
323	1.1	1	4.8	0.0
324	0.3	2	4.8	3.7
325	0.3	1	4.8	0.0
326	0.6	1	4.8	0.7
327	0.7	1	4.8	0.1
328	0.5	2	4.8	0.3
329	0.8	1	4.8	0.2
330	0.7	1	4.8	0.5
331	0.3	3	4.9	1.0
332	0.7	2	4.9	0.5
333	0.4	2	4.9	0.2
334	1.1	3	4.9	0.8
335	0.5	2	4.9	0.4
336	0.7	3	4.9	0.0
337	1.1	2	4.9	0.3

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
338	0.9	2	4.9	0.4
339	0.8	2	4.9	0.5
340	0.9	2	4.9	0.4
341	0.9	2	4.9	0.4
342	0.6	1	4.9	0.1
343	0.7	1	4.9	1.4
344	0.4	1	4.9	0.1
345	1.9	4	4.9	0.9
346	0.6	1	4.9	1.1
347	0.5	1	4.9	0.4
348	0.5	1	4.9	0.9
349	0.6	1	5.0	1.3
350	0.7	5	5.1	0.9
351	0.3	5	5.1	0.3
352	0.4	4	5.1	0.1
353	0.7	3	5.1	0.5
354	0.6	2	5.1	0.2
355	0.6	3	5.1	0.1
356	0.3	2	5.1	1.0
357	1.0	1	5.1	0.3
358	0.5	2	5.1	1.5
359	0.4	1	5.1	0.3
360	0.3	2	6.1	1.2
361	1.0	1	5.1	0.3
362	0.5	1	5.2	1.3
363	0.4	1	5.2	0.7
364	0.5	3	5.2	1.3
365	0.8	2	5.2	0.0
366	0.9	2	5.2	0.2
367	0.5	2	5.2	0.8
368	0.6	2	5.2	0.4
369	0.4	3	5.3	1.3
370	0.6	2	5.3	0.6
371	0.5	1	5.3	0.1
372	0.8	3	5.3	1.3
373	0.4	4	5.3	0.0
374	0.4	3	5.3	0.3
375	0.7	2	5.3	0.2
376	1.2	1	5.3	0.2
377	0.9	2	5.3	0.9
378	0.9	3	5.3	0.9
379	1.2	2	5.3	0.2
380	0.9	2	5.3	0.3

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
381	1.2	1	5.4	0.7
382	0.5	2	5.4	4.7
383	0.4	1	5.4	0.6
384	0.3	1	5.5	1.0
385	0.6	1	5.5	2.4
386	0.4	1	5.5	2.0
387	0.7	1	5.5	1.2
388	0.8	2	5.6	1.0
389	0.6	1	5.6	0.5
390	0.5	4	5.6	1.4
391	0.2	3	5.6	1.1
392	1.1	2	5.6	0.1
393	0.6	2	5.6	0.2
394	0.9	2	5.6	1.1
395	2.6	1	5.7	1.4
396	0.6	1	5.7	4.3
397	1.1	2	5.8	1.2
398	0.8	1	5.8	1.9
399	0.6	2	5.8	2.0
400	0.8	1	5.8	0.3
401	0.7	1	5.9	3.0
402	0.4	3	5.9	1.9
403	0.3	3	5.9	0.5
404	0.8	2	5.9	0.0
405	0.6	2	5.9	0.2
406	1.1	1	5.9	1.2
407	0.3	6	6.0	2.1
408	0.3	5	6.0	0.7
409	0.3	4	6.0	0.0
410	0.3	3	6.0	0.0
411	0.5	2	6.0	0.4
412	0.5	1	6.0	0.0
413	0.4	1	6.0	1.2
414	0.2	2	6.0	0.4
415	0.3	1	6.0	0.0
416	0.3	3	6.0	1.2
417	1.2	2	6.0	0.1
418	2.2	1	6.0	0.2
419	0.5	1	6.1	2.2
420	0.4	2	6.1	1.3
421	0.4	1	6.1	0.0
422	0.8	1	6.1	1.1
423	0.4	1	6.1	1.6

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
424	0.8	1	6.1	0.9
425	0.3	1	6.1	0.6
426	1.0	3	6.2	3.2
427	1.4	3	6.2	0.8
428	0.9	2	6.2	0.7
429	1.1	1	6.3	0.1
430	1.3	1	6.3	3.0
431	1.1	5	6.3	1.3
432	1.4	4	6.3	0.5
433	0.5	4	6.3	0.0
434	0.4	3	6.3	0.2
435	0.5	2	6.3	0.5
436	0.6	1	6.3	0.7
437	0.3	3	6.5	8.3
438	0.8	3	6.5	0.6
439	0.7	3	6.5	0.0
440	4.7	2	6.5	0.3
441	1.0	8	6.6	0.1
442	1.0	7	6.6	1.6
443	0.4	7	6.6	0.1
444	0.4	6	6.6	0.9
445	0.4	8	6.6	2.0
446	0.6	7	6.6	0.1
447	0.2	6	6.6	0.0
448	0.4	5	6.6	0.2
449	1.0	4	6.6	0.9
450	0.4	4	6.6	0.8
451	0.5	3	6.6	0.1
452	0.4	2	6.6	0.1
453	0.5	1	6.6	1.8
454	0.4	3	6.7	1.8
455	0.4	3	6.7	0.1
456	0.5	2	6.7	0.3
457	0.2	2	6.7	0.8
458	0.6	2	6.7	0.6
459	0.6	3	6.7	0.5
460	0.8	2	6.7	0.5
461	0.3	2	6.7	0.0
462	0.7	2	6.7	1.0
463	0.3	3	6.7	0.6
464	0.3	2	6.7	0.9
465	0.6	1	6.7	0.0
466	0.8	2	6.8	1.6

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
467	0.8	2	6.8	0.1
468	0.3	4	6.8	1.2
469	0.3	3	6.8	0.9
470	0.1	2	6.8	0.3
471	0.6	1	6.8	0.1
472	0.6	1	6.8	1.4
473	0.5	1	6.8	0.3
474	0.6	1	6.9	2.0
475	0.6	4	7.1	0.0
476	0.8	3	7.1	0.0
477	1.0	3	7.1	0.0
478	0.9	3	7.1	0.0
479	0.8	2	7.1	0.7
480	0.5	3	7.1	0.3
481	0.9	2	7.2	0.5
482	1.1	1	7.2	1.0
483	0.9	1	7.2	0.9
484	0.6	1	7.2	0.5
485	0.5	1	7.2	0.5
486	0.7	1	7.2	1.1
487	0.5	1	7.2	1.0
488	0.4	1	7.2	1.0
489	0.4	1	7.3	1.7
490	0.8	4	7.3	1.4
491	0.4	4	7.3	0.3
492	1.2	3	7.3	0.1
493	0.7	2	7.3	0.4
494	1.3	1	7.3	0.7
495	0.8	3	7.4	2.2
496	0.4	1	7.4	1.1
497	0.9	1	7.4	0.6
498	1.3	1	7.4	1.0
499	0.9	2	7.5	2.4
500	0.3	1	7.5	1.5
501	0.8	1	7.5	3.0
502	0.8	1	7.5	1.4
503	0.3	1	7.6	3.8
504	1.3	1	7.6	1.0
505	1.1	2	7.6	0.0
506	0.4	1	7.6	0.6
507	0.8	1	7.7	7.2
508	0.3	1	7.7	0.5

Table 1. (Cont.)

Observation Number	Service Time (Minutes)	Queue Length	Time of Day (7:00 A.M. is 0.0)	Time Between Arrivals (Minutes)
509	1.4	1	7.8	0.8
510	0.4	1	7.8	3.7
511	0.4	1	7.9	2.0
512	1.2	1	7.9	2.5
513	0.7	1	8.0	1.4
514	1.5	2	8.0	2.1
515	1.5	2	8.0	0.0
516	0.6	1	8.0	2.6
517	0.4	1	8.0	0.4
518	1.0	1	8.2	9.5
519	1.9	1	8.2	1.0
520	0.3	1	8.3	7.4
521	0.7	1	8.4	4.6
522	0.6	1	8.6	7.5

APPENDIX III

Table 2. "F" Test Results of Correlation Between Service Time and Queue Length and Between Service Time and Time of Day

Time Period (0.0 is 7:00 A.M.)	Component	Service Time vs Queue Length		Service Time vs Time of Day	
		"F" Ratio	Significance Level	"F" Ratio	Significance Level
0.0 - 0.9	Linear	1.56	--	0.03	--
	Quadratic	2.05	--	2.06	--
	Cubic	0.04	--	1.39	--
	Quartic	0.66	--	5.24	0.025
1.0 - 1.9	Linear	0.01	--	1.05	--
	Quadratic	0.00	--	4.05	0.05
	Cubic	3.24	--	1.46	--
	Quartic	0.04	--	0.24	--
2.0 - 2.9	Linear	9.80	0.01	4.01	0.05
	Quadratic	1.27	--	0.08	--
	Cubic	1.03	--	10.73	0.01
	Quartic	0.00	--	0.92	--
3.0 - 3.9	Linear	2.61	--	3.01	--
	Quadratic	0.65	--	0.23	--
	Cubic	0.14	--	0.05	--
	Quartic	0.01	--	1.16	--
4.0 - 4.9	Linear	1.55	--	8.34	0.01
	Quadratic	4.01	--	15.03	0.01
	Cubic	0.14	--	0.00	--
	Quartic	0.00	--	0.00	--

Table 2. (Cont.)

Time Period (0.0 is 7:00 A.M.)	Component	Service Time vs Queue Length "F" Ratio	Service Time vs Queue Length Significance Level	Service Time vs Time of Day "F" Ratio	Service Time vs Time of Day Significance Level
5.0 - 5.9	Linear	6.23	0.025	2.13	--
	Quadratic	0.12	--	0.53	--
	Cubic	0.18	--	0.48	--
	Quartic	0.07	--	1.96	--
6.0 - 6.9	Linear	0.42	--	0.25	--
	Quadratic	0.16	--	4.54	0.05
	Cubic	0.39		0.58	--
	Quartic	0.06	--	2.89	--
7.0 - 7.9	Linear	0.02	--	0.25	--
	Quadratic	2.16	--	0.08	--
	Cubic	0.02	--	0.06	--
	Quartic	0.00	--	0.00	--
8.0 - 8.6	Linear	2.05	--	0.21	--
	Quadratic	0.00	--	0.51	--
	Cubic	0.00	--	1.72	--
	Quartic	0.00	--	2.04	--
0.0 - 2.9	Linear	3.19	--	15.94	0.01
	Quadratic	1.19	--	4.44	0.05
	Cubic	0.99	--	8.91	0.01
	Quartic	0.04	--	1.37	--
3.0 - 5.9	Linear	3.81	--	0.00	--
	Quadratic	0.38	--	0.49	--
	Cubic	0.62	--	1.60	--
	Quartic	0.18	--	3.11	--

Table 2. (Cont.)

Time Period (0.0 is 7:00 A.M.)	Component	Service Time vs Queue Length "F" Ratio	Service Time vs Queue Length Significance Level	Service Time vs Time of Day "F" Ratio	Service Time vs Time of Day Significance Level
6.0 - 8.6	Linear	1.17	--	1.21	--
	Quadratic	0.02	--	0.08	--
	Cubic	1.25	--	0.05	--
	Quartic	0.16	--	2.81	--
All Day (Even)	Linear	2.39	--	19.77	0.01
	Quadratic	1.40	--	5.62	0.01
	Cubic	0.01	--	0.29	--
	Quartic	0.63		3.63	--
All Day (Odd)	Linear	4.01	0.05	10.39	0.01
	Quadratic	1.10	--	9.66	0.01
	Cubic	0.09	--	1.12	--
	Quartic	0.10	--	0.09	--

APPENDIX IV

MINIMUM COST SOLUTION. (7:00 A. M. to 3:45 P. M.)

Description of Model:

1. Poisson arrivals
2. Exponential service
3. Infinite queue
4. Multiple channels

Known:

1. Mean arrival rate (λ) = 1.22 arrivals per minute (actual)
2. Mean service rate (μ) = 1.20 served per minute (actual)
3. Number of channels = number of clerks = M
4. Utilization factor = ρ = $\lambda / M \mu$
5. Mean waiting time, total = W = L/λ
6. Total number in system, average = L = $L_q + \rho M$
7. Mean number in queue = $L_q = \rho\, e_m(\rho M)/(1-\rho)D_{m-1}(\rho M)$

Solution for Mean Waiting Time (varying number of clerks):

1. One clerk

λ = 1.22 arrivals per minute

μ = 1.20 served per minute

M = 1 (clerk)

$\rho = \lambda / M \mu$

$e_m(\rho M) = e_m(1.02 \times 1) = e_m(1.02) = 0.367$

Using Table V, Morse, <u>Queues, Maintenance, and Inventories</u>.

APPENDIX IV (Cont.)

Solution for Mean Waiting Time (Cont.)

$D_{m-1}(\rho M) = D_o(\rho M) = 0.339$

Using Table V, Morse, <u>Queues, Maintenance, and Inventories</u>.

$L_q = (1.02)(0.367)/(1-1.02)(0.339) = \infty$

$L = \infty$

$W = \infty$

2. Two clerks

$\lambda = 1.22$ arrivals per minute

$\mu = 1.20$ served per minute

$M = 2$ (clerks)

$\rho = \lambda/M\mu = 1.22/2\ (1.20) = 0.51$

$e_m(\rho M) = e_2\ (0.51 \times 2) = e_2\ (1.02) = 0.18$

$D_{m-1}(\rho M) = D_1(\rho M) = 0.56$

$L_q = (0.51)(0.18)/(0.49)(0.56) = 0.091\ /0.274 = 0.33$

$L = L_q + \rho M = 0.33 + 1.02 = 1.35$

$W = L/\lambda = 1.35/1.22 = 1.16$ minutes

3. Three clerks

$\lambda = 1.22$ arrivals per minute

$\mu = 1.20$ served per minute

$M = 3$ (clerks)

$\rho = \lambda/M\mu = 1.22/3(1.20) = 1.22/3.60 = 0.34$

$e_m\ (\rho M) = e_3(0.34 \times 3) = e_3(1.02) = 0.056$

$D_{m-1}(\rho M) = D_2(0.34 \times 3) = D_3(1.02) = 0.66$

APPENDIX IV (Cont.)

Solution for Mean Waiting Time (Cont.)

$$L_q = (0.34)(0.056)/(0.66)(0.66) = 0.019/0.436 = 0.044$$

$$L = L_q + \rho M = 0.044 + 0.34(3) = 0.044 + 1.02 = 1.064$$

$$W = L/\lambda = 1.064/1.22 = 0.86 \text{ minutes}$$

Solution for Minimum Cost 7:00 A. M. to 3:45 P. M.:

1. One clerk

$$W = \infty$$

Total cost of Operation per Day(C) = Cost of Service(S) + Cost of Waiting(W)

Cost of Service(S) = Number of Clerks x Cost/Clerk/Day (dollars)

Cost of Waiting(W) = Number Served/Day x Mean Wait/Service (minutes) x Cost/Minute (dollars)

$$C_1 = S + W$$

$$C_1 = 1 \text{ Clerk} \times \$15.20/\text{Day} + 522 \text{ Served} \times \infty \text{ Wait} \times \$0.039/\text{Minute}$$

$$C_1 = \infty$$

2. Two clerks

$$W = 1.16 \text{ minutes}$$

$$C_2 = 2 \times 15.20 + 522 \times 1.16 \times \$0.039$$

$$C_2 = \$30.40 + \$23.49$$

$$C_2 = \$53.89$$

APPENDIX IV (Cont.)

Solution for Minimum Cost (Cont.)

3. Three clerks

$$W = 0.86 \text{ minutes}$$

$$C_3 = 3 \times \$15.20 + 522 \times 0.86 \times \$0.039$$

$$C_3 = \$45.60 + \$17.75$$

$$C_3 = \$63.35$$

<u>Two clerks = minimum cost solution 7:00 A. M. to 3:45 P. M.</u>

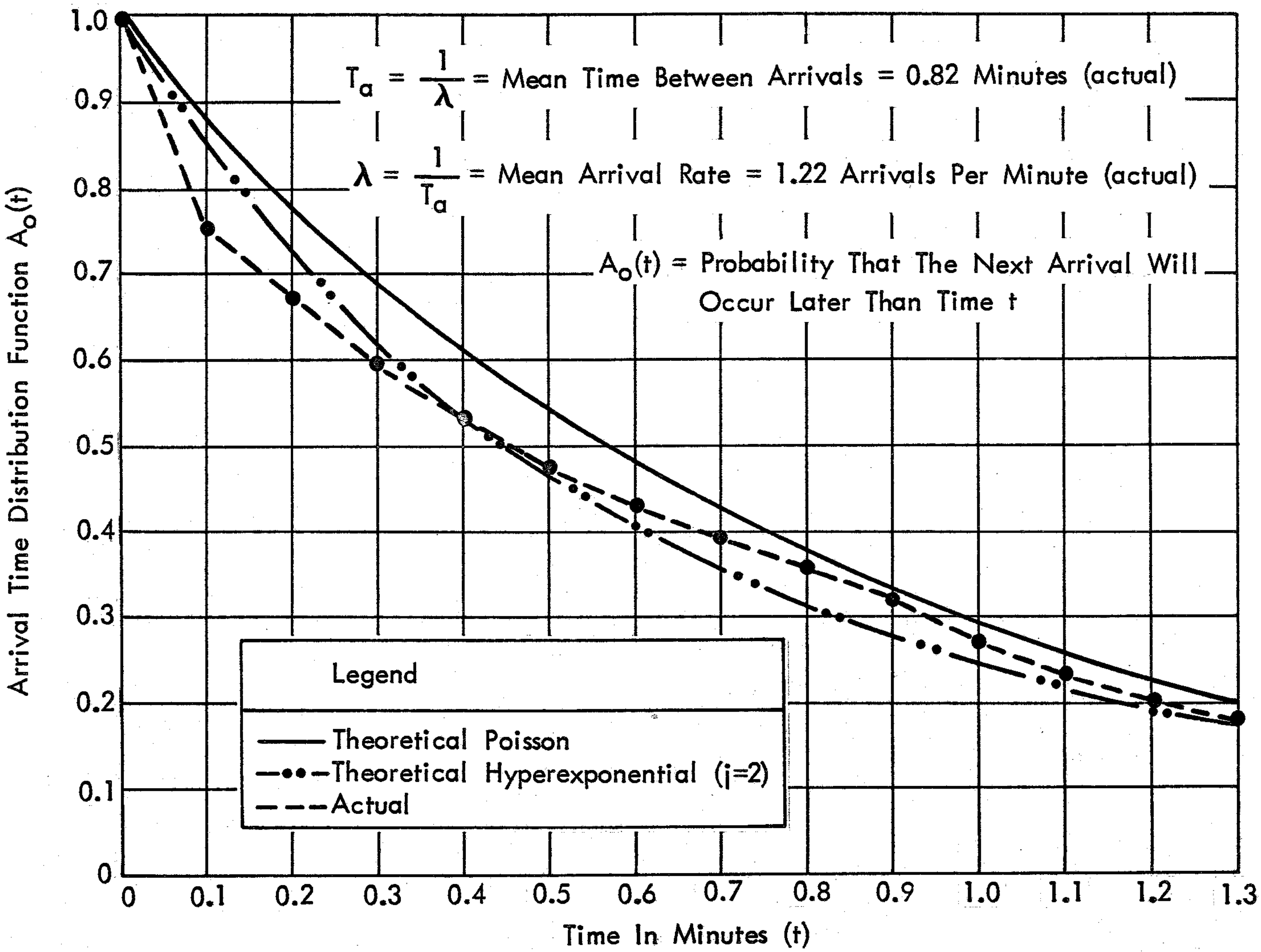

Figure 5 Arrival Time Distribution For Period Between 7:00 A.M. And 3:45 P.M.

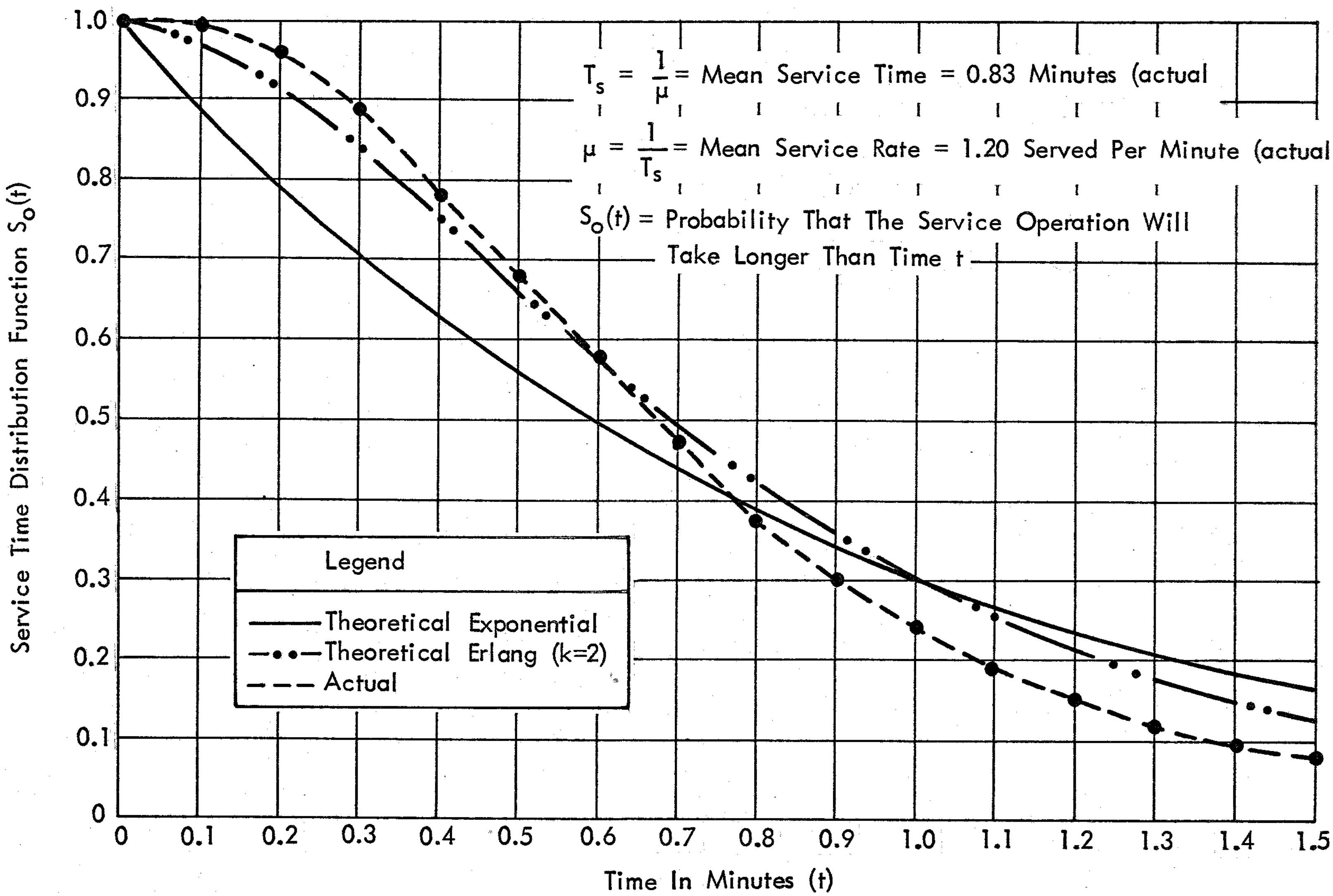

Figure 6 Service Time Distribution For Period Between 7:00 A.M. And 3:45 P.M.

APPENDIX IV (Cont.)

MINIMUM COST SOLUTION. (7:00 A. M. to 8:00 A. M.)

Description of Model:

1. Poisson arrivals
2. Exponential service
3. Infinite queue
4. Multiple channels

Known:

1. $\lambda = 2.28$ arrivals per minute
2. $\mu = 0.90$ served per minute
3. $\rho = \lambda / M \mu$
4. $W = L / \lambda$
5. $L = L_q + \rho M$
6. $L_q = \rho e_m(\rho M)/(1 - \rho) D_{m-1}(\rho M)$

Solution for Mean Waiting Time:

1. One clerk

$\rho = 2.28(1)(0.90) = 2.53$

$W = \infty$ (because ρ is greater than 1)

2. Two clerks

$\rho = 2.28/(2)(0.90) = 1.27$

$W = \infty$ (because ρ is greater than 1)

APPENDIX IV (Cont.)

Solution for Mean Waiting Time: (Cont.)

3. Three clerks

$\rho = 2.28/(3)(0.90) = 0.84$

$e_m(\rho M) = e_3(\rho M) = 0.215$

$D_{m-1}(\rho M) = D_2(\rho M) = 0.35$

$L_q = (0.84)(0.215)/(0.16)(0.35) = 0.18/0.056$

$L_q = 3.21$

$L = 3.21 + 0.84(3) = 3.21 + 2.52$

$L = 5.73$

$W = 5.73/2.28 = 2.07$ minutes

4. Four clerks

$\rho = 2.28/4(0.90) = 0.63$

$e_m(\rho M) = e_5(\rho M) = 0.135$

$D_{m-1}(\rho M) = D_4(\rho M) = 0.52$

$L_q = (0.63)(0.135)/(0.37)(0.41) = 0.085/0.152$

$L_q = 0.56$

$L = 0.56 + 0.63(4)$

$L = 3.08$

$W = 3.08/2.28 = 1.35$ minutes

5. Five clerks

$\rho = 2.28/5(0.90) = 0.51$

$e_m(\rho M) = e_5(\rho M) = 0.07$

$D_{m-1}(\rho M) = D_4(\rho M) = 0.52$

APPENDIX IV (Cont.)

Solution for Mean Waiting Time: (Cont.)

$$L_q = (0.51)(0.07)/(0.49)(0.52) = 0.036/0.255$$

$$L_q = 0.14$$

$$L = 0.14 + 0.51(5)$$

$$L = 2.69$$

$$W = 2.69/2.28 = 1.18 \text{ minutes}$$

Solution for Minimum Cost:

1. One clerk and two clerks

$$C_1 \ \& \ C_2 = \infty \quad (\text{because } \rho \text{ is greater than } 1)$$

2. Three clerks

$$W = 2.07 \text{ minutes}$$

$$C_3 = 3 \text{ clerks} \times 1 \text{ hour} \times \$1.90/\text{hour} + 133 \times 2.07 \times \$0.039$$

$$C_3 = \$5.70 + \$10.77$$

$$C_3 = \$16.47$$

3. Four clerks

$$W = 1.35 \text{ minutes}$$

$$C_4 = 4 \times 1 \times \$1.90 + 133 \times 1.35 \times \$0.039$$

$$C_4 = \$7.60 + \$7.05$$

$$C_4 = \$14.65$$

4. Five clerks

$$W = 1.18 \text{ minutes}$$

$$C_5 = 5 \times 1 \times \$1.90 + 1.18 \times \$0.039$$

APPENDIX IV (Cont.)

Solution for Minimum Cost: (Cont.)

$C_5 = \$9.50 + \$6.12$

$C_5 = \$15.62$

<u>Four clerks = minimum cost solution from 7:00 A. M. to 8:00 A. M.</u>

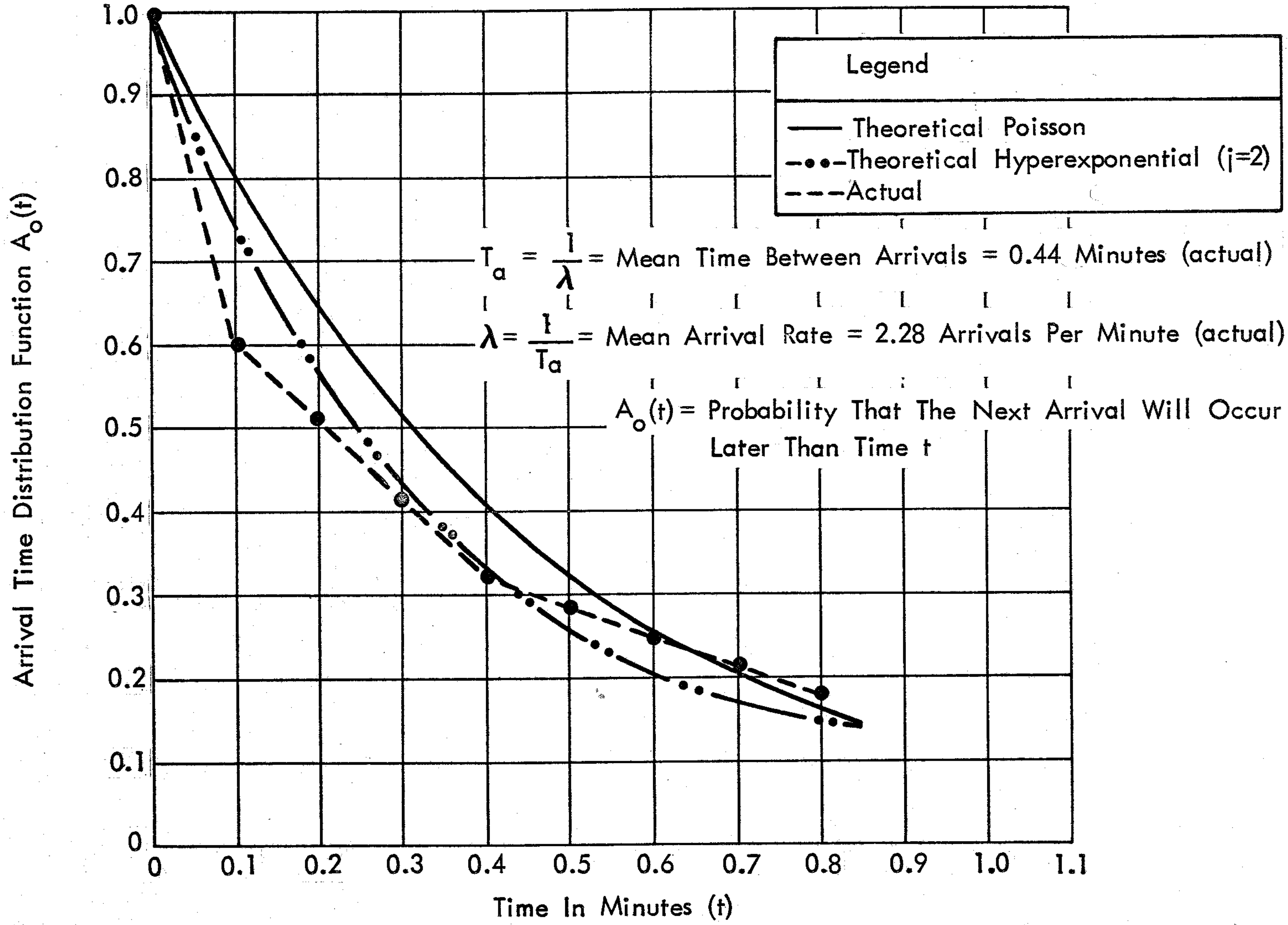

Figure 7 Arrival Time Distribution For Period Between 7:00 A.M. And 8:00 A.M.

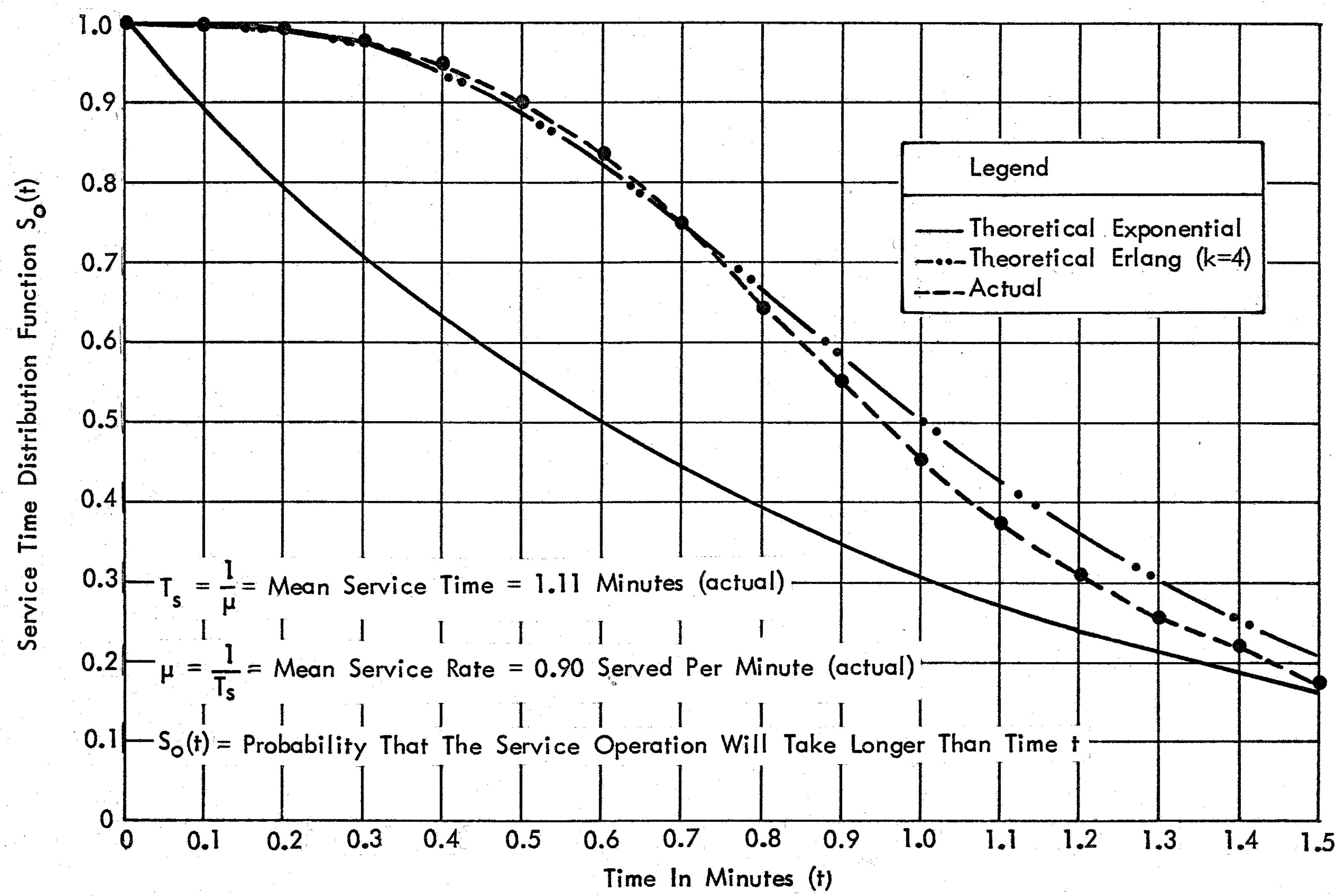

Figure 8 Service Time Distribution For Period Between 7:00 A.M. And 8:00 A.M

APPENDIX IV (Cont.)

MINIMUM COST SOLUTION. (8:00 A. M. to 3:45 P. M.)

Description of Model:

1. Poisson arrivals
2. Exponential service
3. Infinite queue
4. Multiple channels

Known:

1. $\lambda = 1.04$ arrivals per minute
2. $\mu = 1.37$ served per minute
3. $\rho = \lambda / M\mu$
4. $W = L/\lambda$
5. $L = L_q + \rho M$
6. $L_q = \rho\, e_m(\rho M)/(1 - \rho)\, D_{m-1}(\rho M)$

Solution for Mean Waiting Time:

1. One clerk

$$\rho = 1.04/(1)\,1.37 = 0.76$$

$$e_m(\rho M) = e_1(\rho M) = 0.35$$

$$D_{m-1}(\rho M) = D_o(\rho M) = 0.475$$

$$L_q = (0.76)(0.35)/(0.24)(0.475) = 0.266/0.114$$

$$L_q = 2.33$$

$$L = 2.33 + 0.76(1)$$

$$L = 3.09$$

$$W = 3.09/1.04 = 2.97 \text{ minutes}$$

APPENDIX IV (Cont.)

Solution for Mean Waiting Time: (Cont.)

2. Two clerks

ρ = 1.04/(2) 1.37 = 0.38

$e_2(\rho M) = e_2(\rho M) = 0.14$

$D_{m-1}(\rho M) = D_1(\rho M) = 0.65$

L_q = (0.38)(0.14)/(0.62)(0.65) = 0.053/0.403

L_q = 0.13

L = 0.13 + 0.38(2)

L = 0.89

W = 0.89/1.04 = 0.85 minutes

3. Three clerks

ρ = 1.04/(3) 1.37 = 0.25

$e_m(\rho M) = e_3(\rho M) = 0.125$

$D_{m-1}(\rho M) = D_2(\rho M) = 0.75$

L_q = (0.25)(0.125)/(0.75)(0.75)(0.75) = 0.031/0.552

L_q = 0.056

L = 0.056 + 0.25(3)

L = 0.81

W = 0.81/1.04 = 0.78 minutes

Solution for Minimum Cost 8:00 A. M. to 3:45 P. M.:

1. One clerk

W = 2.97 minutes

C_1 = One clerk x 7 hours/day x \$1.90/hour + 389 waiting x 2.97 minutes/wait x \$0.039/minute

APPENDIX IV (Cont.)

Solution for Minimum Cost: (Cont.)

$C_1 = \$13.30 = \$45.12$

$C_1 = \$58.42$

2. Two clerks

$W = 0.85$ minutes

$C_2 = 2 \times 7 \times \$1.90 + 389 \times 0.85 \times \$0.039$

$C_2 = \$26.60 + \$12.78$

$C_2 = \$39.38$

3. Three clerks

$W = 0.78$ minutes

$C_3 = 3 \times 7 \times \$1.90 + 389 \times 0.78 \times \$0.039$

$C_3 = \$39.90 + \$11.62$

$C_3 = \$51.52$

<u>Two clerks = minimum cost solution from 8:00 A. M. to 3:45 P. M.</u>

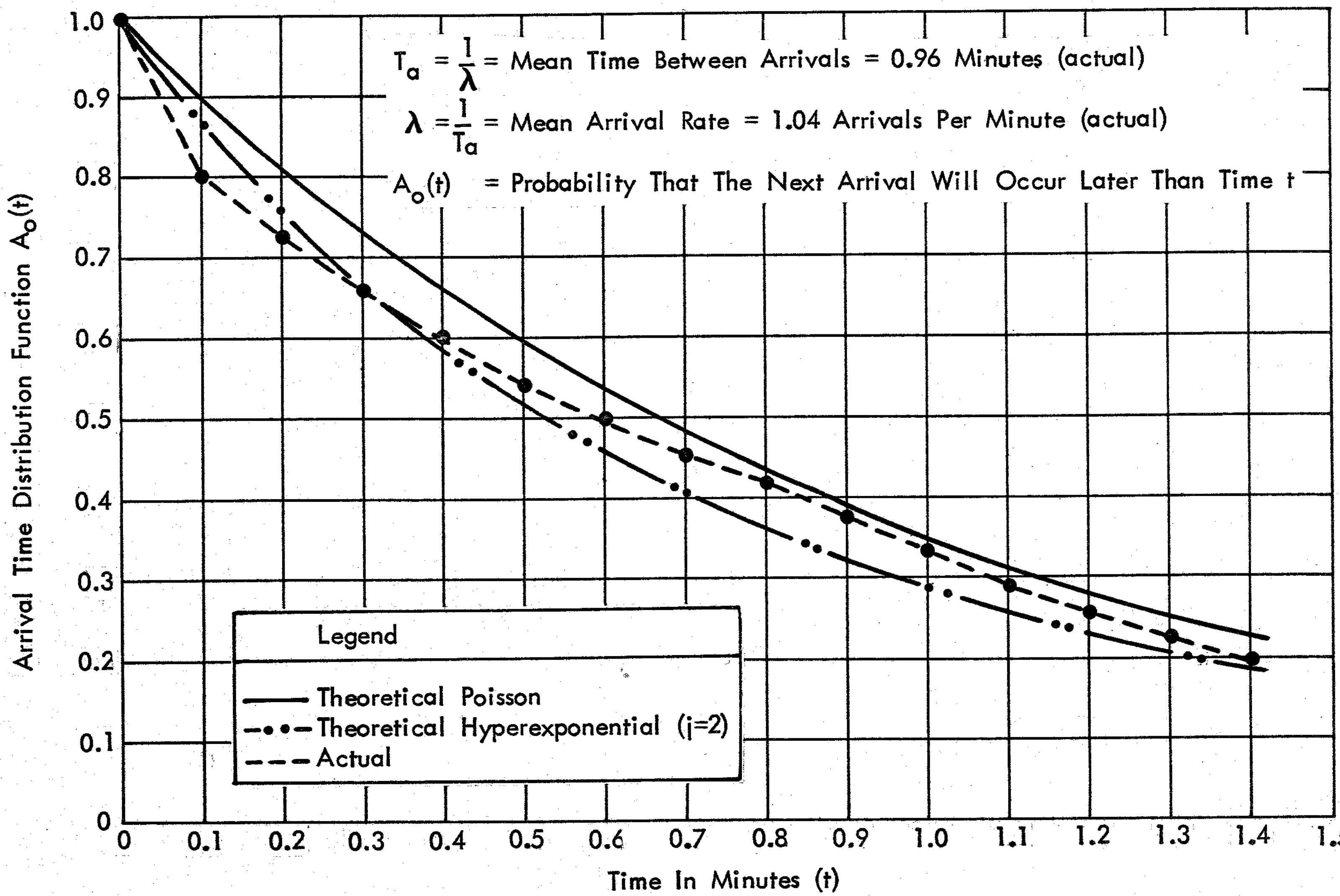

Figure 9 Arrival Time Distribution For Period Between 8:00 A.M. And 3:45 P.M.

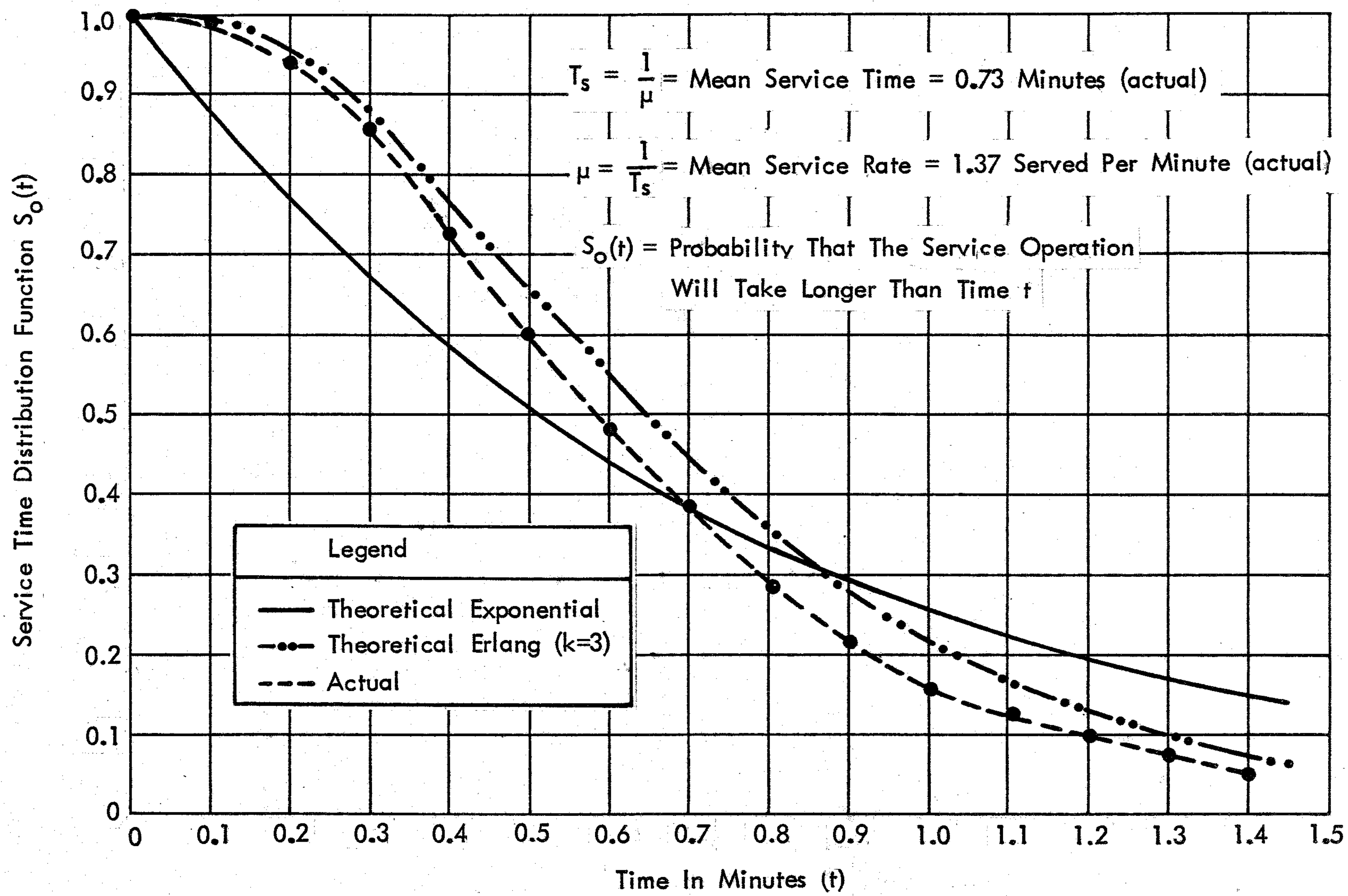

Figure 10 Service Time Distribution For Period Between 8:00 A.M. And 3:45 P.M.

APPENDIX IV (Cont.)

Cost Comparison:

1. Existing System (3 clerks all day)

   Total Cost Existing System = C_3 (7:00 A. M. to 8:00 A. M.)

   + C_3 (8:00 A. M. to 3:45 P. M.)

   Total Cost = \$16.47 + \$51.52 = \$67.99

2. Minimum Cost System (Considering Effects of Time of Day on Solution) Utilizing 4 clerks from 7:00 A. M. to 8:00 A. M. and 2 clerks from 8:00 A. M. to 3:45 P. M.

   Total Minimum Cost = C_4 (7:00 A. M. to 8:00 A. M.)

   + C_2 (8:00 A. M. to 3:45 P. M.)

   Total Minimum Cost = \$14.65 = \$39.38 = \$54.03

3. Minimum Cost System (Not Considering Effects of Time of Day on Solution)

   This solution not calculated for reasons discussed in Chapter IV. Theoretically the cost would be infinite. Practically the cost will exceed by a large margin both alternatives calculated above.

4. Reduction in Cost of Operation

   Reduction in Cost per Day = Cost of Existing System per Day

   - Cost of Minimum Cost System per Day

   Reduction in Cost per Day = \$67.99 - \$54.03 = \$13.96/Day

APPENDIX IV (Cont.)

Cost Comparison: (Cont.)

Reduction in Cost per year = Savings/Day x Number Working Days/Year

Reduction in Cost per Year = $13.96 x 260 = $3629.60

APPENDIX V

Item		Observation Sequence								
		1	2	3	4	5	6	7	8	9
(A)	Arrival Time	0.0	0.1	0.1	0.8	1.0	1.5	3.2	4.0	4.3
(B)	Begin Service	4.0	4.1	4.9	4.9	5.6	5.6	6.0	6.0	6.2
(C)	Waiting Time (A–B)	4.0	4.0	4.8	4.1	4.6	4.1	2.8	2.0	1.9
(D)	End Service	5.6	5.6	6.0	6.0	6.2	6.5	6.5	6.7	6.7
(E)	Service Time (B–D)	1.6	1.5	1.1	1.1	0.6	0.9	0.5	0.7	0.5
(F)	Time Between Arrivals	0.0	0.1	0.0	0.7	0.2	0.5	1.7	0.8	0.3
(A)	Arrival Time	4.5	4.8	6.0	6.1	6.2	6.3	6.4	7.5	7.6
(B)	Begin Service	6.5	6.7	6.7	7.2	7.8	8.0	9.5	9.7	10.0
(C)	Waiting Time	2.0	1.9	0.7	1.1	1.6	1.7	3.1	2.2	2.4
(D)	End Service	7.2	7.8	8.0	9.0	9.7	10.0	10.2	10.6	11.3
(E)	Service Time	0.7	1.1	1.3	1.8	1.9	2.0	0.7	0.9	1.3
(F)	Time BetweenArrival	0.2	0.3	1.2	0.1	0.1	0.1	0.1	1.1	0.1
(A)	Arrival Time	7.8	8.0	8.3	9.0	9.5	10.6	10.6	11.5	11.8
(B)	Begin Service	10.2	11.3	11.6	11.8	12.4	12.8	14.3	14.4	14.6
(C)	Waiting Time	2.4	3.3	3.3	2.8	2.9	2.2	3.7	2.9	2.8
(D)	End Service	11.6	11.8	12.4	12.8	14.3	14.4	14.6	15.1	15.6
(E)	Service Time	1.4	0.5	0.8	1.0	1.9	1.6	0.3	0.7	1.0
(F)	Time Between Arrival	0.2	0.2	0.3	0.7	0.5	1.1	0.0	0.9	0.3

Figure 11 Sample Data Collection Form

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